A steady-state technique for local heat-transfer measurement and its application to the flat plate

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A steady-state technique of heat-transfer measurement has been developed based on the method of Seban, Emery & Levy (1959) whereby energy is dissipated by the Joule effect in a thin metal sheet on the surface of a model. For the present application, use was made of very thin but mechanically resistant films of metal of very nearly constant thickness, obtained by a simple mirror-silvering technique. The present investigation was prompted by the desire to make very local measurements of heat transfer for application in regions where large variations in convective heat flux and therefore in temperature could be expected.

Comparison between theory and experiment has been made in the simple case of a flat plate with constant heat flux for which a rigorous computation could be made based on the theory of Chapman & Rubesin (1949). The model was so conceived that the heat losses were small enough to be neglected. Therefore no corrections, which are often inaccurate, were needed for the experimental results, contrary to what is generally done when using other techniques for heat-transfer measurements. The excellent agreement between theory and experiment gives complete confidence in the method. The theoretical analysis showed that the measurements are simply related to the results that could be obtained in the case of an isothermal surface, because of the constant ratio that exists between the corresponding heat-transfer coefficients.

1. Introduction

All the standard methods of heat-transfer measurement have their own advantages and disadvantages. There is, however, one common drawback to all of them which may appear in special applications, i.e. the difficulty of measuring very local heat-transfer rates. The problem presented itself to the author in an investigation made on reattaching supersonic flows where available methods could not be used to detect the local effects on the heat-transfer rate of three-dimensional flow perturbations previously observed (Ginoux 1961). These flow perturbations exhibited wavelengths of the order of a few millimetres and therefore it was required to have heat meters whose dimensions were an order of magnitude smaller. For this reason it was decided to develop a particular steady-state technique of heat-transfer measurement, which would consist in dissipating heat uniformly by the Joule effect at the surface of the models, in very thin films of metal of constant thickness, the insulating backing material being itself of small thickness, so as to minimize transverse heat conduction in the models.

The originality of the method lies more in the use and manufacture of extremely thin films than in its principle. Indeed, the idea of dissipating heat uniformly at the surface of the model has already been used by Seban *et al.* (1959) in low-speed tests. They used nichrome ribbons 0·002 in. thick, glued to a flat surface. However, these ribbons were still too thick for the present investigation and in addition, difficulties were found in gluing them properly, especially on hollow models and on curved surfaces. In the early part of the present study, thin films of constant thickness were formed by evaporation of metal under vacuum but the method was given up because of the fragility of the films and also because the process was extremely lengthy. Later a mirror-silvering technique was used which appeared very satisfactory in all respects.

As a first step in the study, it was decided to apply this technique to a simple type of flow, in order to submit it to a rigorous test. The present article deals with a description of the method and its application to the flow over a flat plate.

The experimental investigation was made in one of the supersonic tunnels of the Training Centre for Experimental Aerodynamics (TCEA) in Rhode-Saint-Genèse, Belgium.

2. Principle of the technique

The heat flux per unit area and unit time (q) is determined from the measured voltage and current and from the total area of the heating element. Temperatures are measured by thermocouples located at the model surface for power-off (T_{wa}) and power-on (T_w) conditions. The heat-transfer coefficient is then computed from the following relationship

$$h_q = q/(T_w - T_{wa}),$$

where subscript q refers to constant heat flux.

The method of obtaining thin films of metal is derived from standard techniques for silvering mirrors. It is extremely simple in its use. Models are immersed successively for a few minutes into solutions of stannous chloride and silver nitrate, respectively. By thoroughly cleaning the models, it is easy to obtain a layer of metal having a thickness constant to within better than 10%, as can be checked electrically. By titration of the solutions, it was found that the mean thickness was of the order of 1μ. However, a direct computation of the thickness based on the surface dimensions, its total resistance and the resistivity of the bulk material gave a value which was quite evidently too small. Therefore, the resistivity of the film must be larger than assumed and consequently the effective thickness of the film for transverse heat conduction is expected to be much smaller than 1μ. This is of course favourable in the application of the technique to local heat-transfer measurements.

In the present study, Araldite was used as the backing insulation and surprisingly strong films of silver were obtained inasmuch as sand paper was needed to remove them. The wall temperatures were limited to about 120 °F in order to avoid warping of the models. Other insulating materials have to be used for somewhat higher temperatures. Further description of the technique has been given by Ginoux (1963).

3. Application to the flow around a wedge

With a view to checking the accuracy of the technique, it was applied to the study of supersonic flow along a flat plate for which there was a rigorous solution to the laminar boundary-layer equations. The model configuration, shown in figure 1, was selected in order to match closely the theoretical conditions. It consists of a wedge of small apex angle (2α), followed by an afterbody AA'BB'. The model is placed in a supersonic stream at zero angle of attack in order to minimize the losses by conduction inside the model. Heat is dissipated uniformly at the same rate on both the upper and lower surfaces of the model.

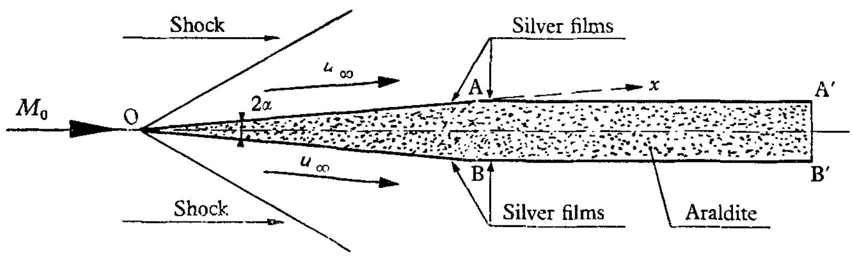

FIGURE 1. Silver-plated wedge model in a supersonic stream.

4. Theory

Chapman & Rubesin (1949) found a rigorous solution for the laminar flow of a compressible fluid over a flat plate, in the case of a polynomial wall-temperature distribution. In particular they obtained the following expression for the heat flux at the wall

$$q = -\tfrac{1}{2}k_\infty T_\infty C_w(u_\infty/\nu_\infty xC)^{\frac{1}{2}}\Sigma a_n(x/L)^n\, Y_n'(0) \qquad (1)$$

for a wall-temperature distribution given by

$$T_w = T_{wa} + T_\infty \Sigma a_n(x/L)^n,$$

where T_{wa} is the adiabatic wall temperature, x the distance along the plate from its leading-edge, L a characteristic length, and the subscript ∞ refers to upstream conditions (figure 1). C is a constant based on a mean value of the wall temperature, $\overline{T}_w$, whereas C_w is a variable coefficient. They are defined by

$$C = \left(\frac{\overline{T}_w}{T_\infty}\right)^{\frac{1}{2}}\frac{T_\infty + S}{\overline{T}_w + S}, \quad C_w = \left(\frac{T_w}{T_\infty}\right)^{\frac{1}{2}}\frac{T_\infty + S}{T_w + S},$$

where S is Sutherland's constant. $Y_n'(0)$ is the value of the function Y_n' computed at the wall, ν the kinematic viscosity, k the thermal conductivity and u the 'undisturbed' mean flow.

In the present study, the increase of the wall temperature above the adiabatic value due to heating was limited for practical reasons to about 80 °F. In these conditions, it is easy to see that $C_w = C = \text{const.}$ to within 1 %. Therefore, if we want to consider the case of constant heat flux (i.e. q = const.), we see from relationship (1) that we have to select $n = \frac{1}{2}$. The corresponding wall-temperature distribution is

$$T_w - T_{wa} = T_\infty a_{\frac{1}{2}} (x/L)^{\frac{1}{2}}, \quad (2)$$

where the coefficient $a_{\frac{1}{2}}$ is given by

$$a_{\frac{1}{2}} = -\frac{2q}{k_\infty T_\infty} \frac{1}{Y'_{\frac{1}{2}}(0)} \left(\frac{\nu_\infty L}{C u_\infty}\right)^{\frac{1}{2}}. \quad (3)$$

From (2) and (3), we can compute the heat-transfer coefficient at constant heat flux

$$h_q = -\tfrac{1}{2} k_\infty Y'_{\frac{1}{2}}(0) \, (C u_\infty / \nu_\infty x)^{\frac{1}{2}}. \quad (4)$$

Chapman & Rubesin gave the value of $Y'_n(0)$ for $n = 0, 1, 2, \ldots$ and for a Prandtl number $\sigma = 0{\cdot}72$. Although these values may be interpolated for intermediate values of n, as suggested by Curle (1962), it was found more accurate to compute directly the coefficient $Y'_{\frac{1}{2}}(0)$ and at the same time evaluate the influence of the Prandtl number. The numerical integration method of Runge–Kutta was used in the calculations. Initial values were given by an exact asymptotic solution for the boundary-layer equations which was easily determined. The steps for the integration were selected small enough to give correct values of $Y'_{\frac{1}{2}}(0)$ to within 1 %. For a Prandtl number of 0·72, it was found that

$$Y'_{\frac{1}{2}}(0) = -0{\cdot}82,$$

while a linear interpolation between $Y'_0(0)$ and $Y'_1(0)$, given by Chapman & Rubesin, led to a value of $-0{\cdot}785$, i.e. smaller by 4 %. Values of $Y'_{\frac{1}{2}}(0)$ obtained for two other values of the Prandtl number are indicated in table 1.

In terms of the Nusselt number, (4) can be written as

$$Nu/\surd R_{ex} = \surd C/2{\cdot}44. \quad (5)$$

5. Comparison between measurements at constant q and T_w

For the flow over an isothermal flat plate, Chapman & Rubesin found that

$$h_T = -\tfrac{1}{2} k_\infty (u_\infty C/\nu_\infty x)^{\frac{1}{2}} \, Y'_0(0), \quad (6)$$

where subscript T refers to a constant wall temperature.

Dividing (4) by (6), we find that

$$h_q/h_T = Y'_0(0)/Y'_{\frac{1}{2}}(0) = G = \text{const.}, \quad (7)$$

i.e. the ratio of the heat-transfer coefficients for constant heat flux and for constant temperature is a constant independent of the Reynolds number and Mach number. Table 1 shows that it is also independent of the Prandtl number in the range $0{\cdot}5 < \sigma < 1{\cdot}0$. This property suggests that the experimental results from the constant-heat-flux technique are simply related to results that could be obtained with the more familiar isothermal method, although it remains to be verified in the presence of a pressure gradient.

The numerical value of G has been evaluated by taking the following approximate expression (Chapman & Rubesin 1949)

$$Y'_0(0) = -\tfrac{1}{2} f''(0) \, \sigma^{\frac{1}{3}} = -0{\cdot}664 \sigma^{\frac{1}{3}},$$

where f is the Blasius function, valid to within 1 % for $0{\cdot}6 < \sigma < 1{\cdot}0$. The results are given in table 1. They show that G is independent of σ within the accuracy of the computation. It is noted that G is very nearly equal to σ for $\sigma = 0{\cdot}72$. This was found first and suggested varying the Prandtl number to the author.

σ	0·50	0·72	1·00
$-Y'_{\frac{1}{2}}(0)$	0·722	0·820	0·918
G	0·730	0·721	0·723

TABLE 1

6. Heat conduction inside the model

The following calculation of the temperature distribution inside an infinite wedge provides a useful measure of heat losses. As shown by relationship (2), the wall temperature varies along the surface of the wedge. Therefore, one may expect surface heat losses due to conduction through the model. They can be easily evaluated if one assumes *a priori* that their effect on the wall-temperature distribution may be neglected so that the boundary condition at the wall is given by equation (2).

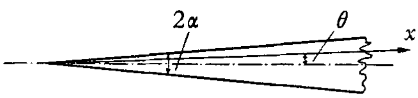

FIGURE 2. Polar co-ordinates for the computation of heat conduction inside the wedge.

Using the polar co-ordinates shown in figure 2 and denoting the apex angle of the wedge by 2α, one has to solve the equation for steady temperature distribution, in polar co-ordinates, namely

$$x\frac{\partial^2 T}{\partial x^2}+\frac{\partial T}{\partial x}+\frac{1}{x}\frac{\partial^2 T}{\partial \theta^2}=0,$$

with the boundary condition

$$T = T_{wa}+T_\infty a_{\frac{1}{2}}(x/L)^{\frac{1}{2}} \quad \text{at} \quad \theta = \pm\alpha.$$

It can easily be seen that the solution is

$$T = T_{wa}+T_\infty a_{\frac{1}{2}}\left(\frac{x}{L}\right)^{\frac{1}{2}}\cos\tfrac{1}{2}\theta/\cos\tfrac{1}{2}\alpha.$$

The heat flux through the surface of the model per unit time and unit area is then

$$q_a = -k_a(\partial T/x\,\partial\theta)_{\theta=\alpha},$$

where k_a is the coefficient of thermal conductivity of the Araldite

($k_a = 0{\cdot}11$ B.Th.U./ft. hr °F for Araldite type D).

q_a is positive when heat goes out of the model. Thus, a good estimate of the ratio of the heat losses to the heat generated at the surface is given by

$$\frac{q_a}{q} = -\frac{k_a}{k_\infty}\frac{1}{Y'_{\frac{1}{2}}(0)}\left(\frac{\nu_\infty}{u_\infty Cx}\right)^{\frac{1}{2}}\tan\tfrac{1}{2}\alpha. \tag{8}$$

7. Model and test conditions

The model was cast with Araldite after correctly positioning the copper-constantan thermocouples inside the mould, then machined in order to bring the thermocouple junctions flush with the surface, and finally silver plated. The thermocouple wires are run spanwise inside the model to minimize the heat losses. Electrodes are installed along the sides of the model to supply the electric power independently to the two surfaces OA and OB (figure 1) of the wedge and to the surfaces AA′ and BB′ of the afterbody. Good electrical contact between the electrodes and the silver layers is ensured by additional silver paint.

The lengths OA and AA′ of the wedge and of the afterbody were both equal to 6 in. and the model span was 7·5 in. Although the wedge angle was rather small (10 deg.) a fairly sharp leading edge could easily be obtained with Araldite. The thickness of the silver films was equal to 1 μ with local variations smaller than $\pm 10\%$.

The model was mounted on a sting and tested in the TCEA 16 in. × 16 in. continuous supersonic wind tunnel $S-1$ at a free-stream Mach number M_0 (figure 1) of 2·21 and at absolute stagnation pressures of 200 and 100 mm Hg, which correspond to free-stream Reynolds numbers based on a length of 1 ft. of 10^6 and $0{\cdot}5 \times 10^6$ respectively. Steady-state conditions were achieved for both power-off and power-on conditions, after approximately 1 hr of running time. The stagnation temperature in the tunnel was maintained at a value close to ambient temperature. The absolute humidity in the tunnel was kept below 10^{-4}.

8. Results and conclusions

Typical results are presented and compared with the theory in figure 3, where the heat-transfer coefficient h and the recovery factor r are plotted against the distance x from the leading edge and also in figure 4 which gives the quantity $N_u/\sqrt{R_{ex}}$ *vs* x. Squared symbols correspond to a stagnation pressure of 100 mm Hg and circular symbols to a pressure of 200 mm Hg. The theoretical values of h and $N_u/\sqrt{R_{ex}}$ are shown by solid lines and given by relationships (4) and (5). The theoretical value of r is equal to $\sigma^{\frac{1}{2}}$, i.e. 0·848 for $\sigma = 0{\cdot}72$. Schlieren pictures indicated that a laminar boundary layer existed in all the tests.

Figures 3 and 4 show excellent agreement between the theory and the test results. The experimental data are uncorrected. Indeed, it can be seen from (8) that for the present test conditions, the ratio q_a/q was smaller than 1 % for x larger than $\frac{1}{5}$ in., which means that the heat losses by conduction through the Araldite were negligible compared to the heat dissipated at the surface of the model, except near the leading edge. Relation (8) was obtained by assuming an infinite wedge, although this was not the case in the experiments. The effects of the finite length of the model was therefore examined experimentally by comparing the measurements made on the surfaces OA and OB of the model (figure 1), with the surfaces AA′ and BB′ of the afterbody of the model heated or or unheated. No difference was observed except close to A and B as expected.

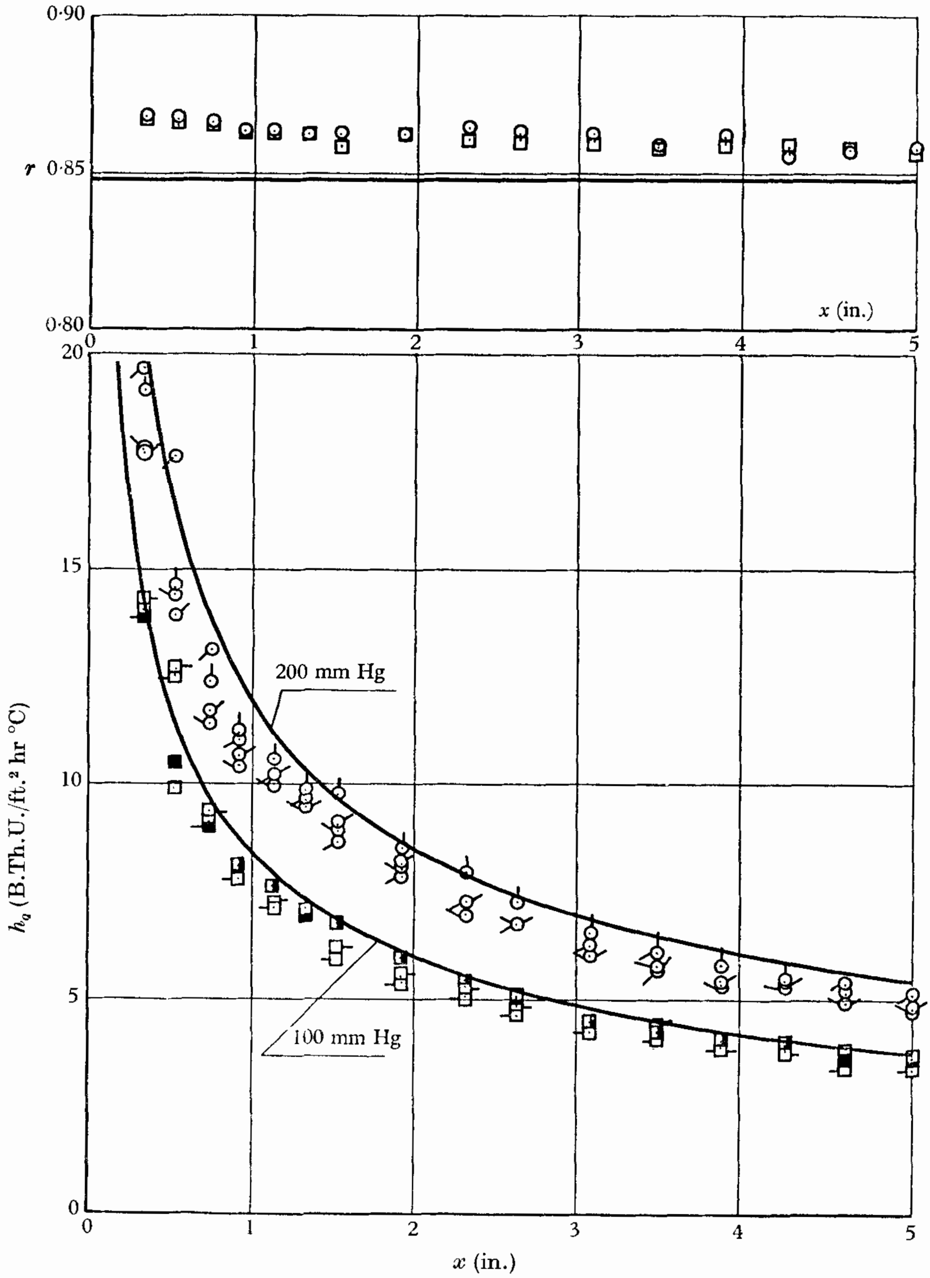

FIGURE 3. Recovery factor and heat-transfer coefficient *vs* x. Squared symbols for stagnation pressure of 100 mm Hg and circular ones for 200 mm Hg. Flags show tests on resilvered model. Unflagged open squares: 5 W; filled squares: 10 W.

It was thus concluded that the heat losses were negligible along most of the surface of the wedge.

Another cause of experimental error is the existence of a non-uniform heat flux. This can be caused either by a variable thickness of the heating elements or by local changes in the electric resistance produced by a variable wall temperature. On the one hand, the thickness of the silver films is never perfectly constant

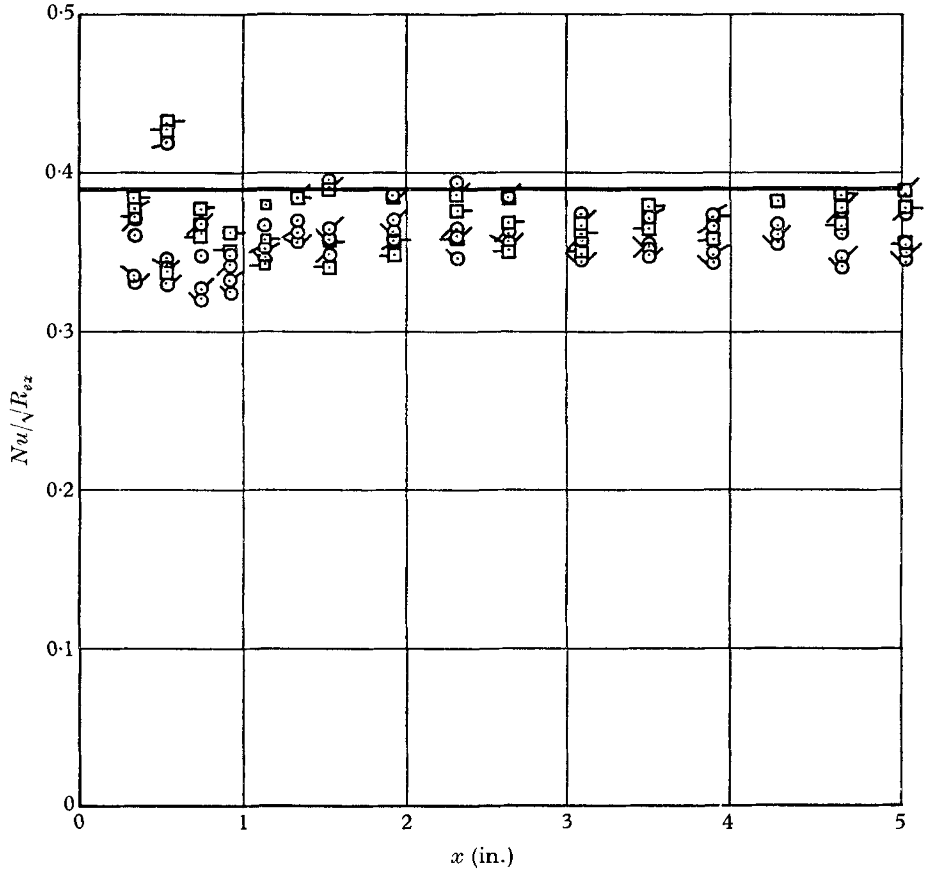

FIGURE 4. Variation of $Nu/\sqrt{R_{ex}}$ with x. Same symbols as in figure 3.

over the whole surface of the model; in the present tests local variations of about 10 % were observed. However, it seems that these variations have a small effect on the results as no significant changes were observed in the measurements after resilvering the model several times, although the imperfections in the films were differently located each time. Flagged symbols are used in figures 3 and 4 to represent different tests on resilvered models. On the other hand, the variations in the wall temperature were purposely kept small in the tests; thus, the electric resistance and consequently the heat flux were not much affected. This was verified experimentally by dissipating two different amounts of power in the heating elements, keeping other conditions unchanged. As seen from figure 3, no systematic difference was observed in the heat-transfer coefficients; unflagged open squares correspond to a power of 5 W dissipated in each surface of the wedge, while filled squares correspond to a power of 10 W.

The mirror-silvering technique allows one to obtain a very thin mechanically resistant film of nearly constant thickness which permits accurate measurements of local heat-transfer rate using Joule heating. Because of the very small thickness of the metallic films, the parasitic heat losses are extremely small and therefore the technique can be used in regions where large surface temperature gradients exist. Of course, the backing material plays an important part in heat losses and it should necessarily have a very low thermal conductivity.

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